Free Space Transmission Loss Measurement of Magneto-Dielectric Materials: Solution Uniqueness and Measurement Tolerance Tradeoffs

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Abstract— **The possibility to determine the specific conductivity, permittivity, and permeability of homogeneous magneto-dielectric materials by measuring it's wideband transmission without measuring it's reflection is investigated. Uniqueness of extracted material parameters along with its sensitivity to measurement tolerances are examined. The novel measurement approach for material samples exhibit very low loss within the test frequency band is introduced.**

I. INTRODUCTION

The RF properties of homogeneous media can be defined through its specific conductivity σ, complex permittivity ε, and permeability μ. A number of non-invasive measurement techniques [1] were developed in that regard. In this paper we investigate the wideband free space transmission loss (TL) only measurement approach where a single/multilayer flat (or nearly flat) material sample is placed between receive and transmit horns and the wideband complex transmission coefficient is measured. A dielectric lens or synthetic aperture [2] might be used to decrease the influence of measurement noise. The material parameters can be found by best-fitting conductive magneto-dielectric (CMD) theoretical model to the measured data. Three problems associated with this particular measurement technique are investigated:

1. Does the wideband free space TL only measurements deliver the unique solution for the sample material parameters, or must the reflection coefficient also be measured?
2. What best-fitting algorithm delivers the unconditional convergence to the right set of material parameters with and without the presence of measurement noise?
3. Can the wideband free space TL only technique be used for samples with very low loss within the test frequency band?

II. UNIQUENESS OF EXTRACTED MATERIAL PARAMETERS BASED ON TRANSMISSION LOSS ONLY MEASUREMENTS

As was noted in [3], even some techniques that simultaneously take into account transmission and reflection characteristics can not uniquely determine constitutive parameters ε, μ and σ. To find out why different material parameters could cause the same transmission and reflection characteristics, let's consider propagation of the plane wave from the half-space 1 (vacuum) to the homogeneous half-space 2 (medium) at the incidence angle θ_1 and the refraction angle θ_2 in accordance with Snell's Law. The relative refraction index n_{21} and the relative wave propagation impedance z_{21} of medium 2 with respect to medium 1 can be defined through constitutive parameters ε, μ and σ of medium 2 and the angular velocity ω of incident wave:

$$\sin\theta_1 / \sin\theta_2 = n_{21}$$

$$n_{21} = \sqrt{\mu_r(1 - j\tan\delta_\mu)\,\varepsilon_r[1 - j(\tan\delta_\varepsilon + \sigma/\omega\varepsilon_0\varepsilon_r)]} \qquad (1)$$

$$z_{21} = \sqrt{\mu_r(1 - j\tan\delta_\mu)\,/\,\varepsilon_r[1 - j(\tan\delta_\varepsilon + \sigma/\omega\varepsilon_0\varepsilon_r)]}$$

where ε_0 is the absolute permittivity of vacuum. Knowing n_{21} and z_{21}, values of ε and μ can be uniquely derived:

$$\varepsilon = n_{21}/z_{21} \qquad \mu = n_{21}z_{21} \qquad (2)$$

As it follows from (1), to distinguish between the $\tan\delta_\varepsilon$ and $\sigma/\omega\varepsilon_0\varepsilon_r$ components of the imaginary part of the relative permittivity ε, the measurements must be taken at a minimum in two frequency points. However, unlike the wideband approach, where measurement errors are averaged over a wide frequency band, any one or two frequency point measurements are very sensitive to measurement tolerances.

According to [4], values of n_{21} and z_{21} can be found from the TE/TM transmission (T) and reflection (R) coefficients along with the normal-to-boundary-surface component of complex wave propagation vector k_{2n} within the medium 2:

$$k_{2n} = k_1\sqrt{n_{21}^2 - \sin^2\theta_1}$$

$$T_{TE} = \frac{2z_{21}\cos\theta_1}{z_{21}\cos\theta_1 + \cos\theta_2} \qquad R_{TE} = \frac{z_{21}\cos\theta_1 - \cos\theta_2}{z_{21}\cos\theta_1 + \cos\theta_2} \qquad (3)$$

$$T_{TM} = \frac{2z_{21}\cos\theta_1}{z_{21}\cos\theta_2 + \cos\theta_1} \qquad R_{TM} = \frac{z_{21}\cos\theta_2 - \cos\theta_1}{z_{21}\cos\theta_2 + \cos\theta_1}$$

where k_1 is the wave propagation number in vacuum.

This case corresponds with the very thick material sample and the only reflection coefficients (RC) can be measured non-invasively. Thus, for the unique determination of n_{21} and z_{21}, both R_{TE} and R_{TM} values should be measured at the oblique incidence. However, during the RC measurement, the PEC plate should be installed instead of the sample to be tested in order to establish the measurement baseline. As is shown in Table I, the material parameters extracted from the RC measurements are very sensitive to the mismatch between the position of the PEC plate and the position of tested sample.

TABLE I
INFLUENCE OF PEC PLATE POSITION MISMATCH FOR HALF-SPACE MEDIA

PEC Mismatch, mil	ε_r	$tan\delta_\varepsilon$ **, 10^{-3}**
No Mismatch	**4.200**	**14.00**
0.5	4.196	34.80
1	4.189	55.59
2	4.165	97.14
10	3.568	444.59

Now let's consider a more practical case where the constitutive parameters are obtained based on the measured transmission and/or reflection characteristics of a flat sample with finite thickness t. The TE/TM reflection R_t and transmission T_t coefficients for this case were derived in [5]:

$$\begin{aligned} T_t &= T_{12}T_{21}e^{-jk_{2n}t} \Big/ (1 - R_{21}^2 e^{-j2k_{2n}t}) \\ R_t &= R_{12} + R_{21}T_{12}T_{21}e^{-j2k_{2n}t} \Big/ (1 - R_{21}^2 e^{-j2k_{2n}t}) \end{aligned} \quad (4)$$

where TE/TM components of the T_{12}, T_{21}, R_{12}, and R_{21} coefficients should be appropriately calculated from (3).

As it follows from (1) – (4), for non-conductive media at normal incidence, the $\varepsilon \leftrightarrow \mu$ swap in material parameters causes 180° phase shift for the RC only. So, for the normal incidence, the measurement of the RC is ultimately needed for the unique determination of material parameters ε and μ.

TABLE III
INFLUENCE OF PEC PLANE POSITION MISMATCH FOR MATERIAL SAMPLE

PEC Mismatch, mil	**Thickness, mil**	ε_r	$tan\delta_\varepsilon$ **, 10^{-3}**
No Mismatch	**33.00**	**4.200**	**14.00**
0.5	33.997	4.118	12.971
1	34.934	4.046	12.496
2	36.853	3.912	11.245
10	50.997	3.275	6.208

The expressions (4) exhibit the oscillatory behavior within the wide frequency band. As it follows from (3) □ (4), the material parameters can be uniquely determined base on the oblique incidence measurement of either TE or TM component of the transmission or reflection coefficients only, since the best-fitting of the periodicity and magnitude of the wideband transmission or reflection curve delivers correct values for the refraction coefficient n_{21}, and the relative wave impedance z_{21}, respectively.

However, as illustrated by the data in Table II, the constitutive parameters found based on the RC measurements, even without presence of the measurement noise are still very sensitive to the position of PEC plate. The TL measurement technique uses a free space TL as a measurement baseline and thus does not have this problem.

This is why only the wideband free space TL measurement technique will be further investigated.

III. CONVERGENCE TO UNIQUE SOLUTION: FINDING GLOBAL OPTIMUM IN PRESENCE OF MEASURED NOISE

The best-fitting procedure that unconditionally converges to the correct set of material parameters is discussed.

A. Global Optimum without Presence of Measurement Noise

The goal function of optimization problem was chosen as a variance between measured and theoretically simulated data over the 2 – 40 GHz frequency band. The case of the measurement data without any systemic and/or random errors was investigated first. Those "measured" data were theoretically simulated based on single layer CMD with the material parameters shown in Table III as an “Exact”.

The sample conductivity was chosen to cause the most absorption loss at lowest frequencies, while having negligible absorption at highest frequencies. The sample thicknesses were chosen to represent two essentially different shapes of transmission loss curves. The 11mil thick sample has almost linear TL characteristics, while the 100mil sample has maxima and minima within the test frequency band.

It was found that even for the single layer CMD and without presence of any measurement noise, the optimization problem has multi-minima and narrow valley nature, with significant tradeoffs between values of thickness, permittivity, and permeability.

It was also found that the consecutive application of GA [6] and DFP [7] algorithms results in the unconditional convergence to the global minimum of goal function and hence, to the right set of material parameters for the single and multilayer CMD. To evaluate and decrease the tolerance of extracted material parameters the solution has to be repeated several times and the statistics should be gathered.

TABLE III
NUMERICAL TOLERANCE OF EXTRACTED MATERIAL PARAMETERS

	t mil	ε_r	$tan\delta_\varepsilon$ **10^{-3}**	σ**, 10^{-2} *Si/m***	μ_r	$tan\delta_\mu$ **10^{-3}**
Exact	**11**	**4.2**	**14**	**5**	**2.5**	**10**
Calc.	11.020 ±0.089	4.195 ±0.026	14.050 ±0.26	4.987 ±0.055	2.498 ±0.014	9.87 ±0.23
Exact	**100**	**4.2**	**14**	**5**	**2.5**	**10**
Calc.	99.995 ±0.12	4.2000 ±0.004	14.001 ±0.34	5.006 ±0.013	2.5002 ±0.003	9.99 ±0.33

The conversion tolerance of the GA and DFP algorithms are shown in Table III for the no measurement noise case. The size of the statistics is 10 and calculated parameters are presented in mean ± standard deviation form. The fact that the mean of material parameters converge much better than might be expected based on its standard deviation values caused by the tradeoffs between the CMD material parameters.

B. *Global Optimum in Presence of Measurement Noise*

The influence of random Gaussian measurement noise on the extracted CMD material parameters is illustrated in Table IV. From the practical prospective, the TL noise standard deviation was varied from 0.1dB to 0.8dB and from 3° to 12°. As seen from Table IV, for a thin material sample with virtually linear TL characteristics, any realistic level of measurement noise prohibits the meaningful extraction of "weak" material parameters such as loss tangents and specific conductivity. For that, the thicker sample with the TL curve minima within the test frequency band is ultimately needed.

TABLE IV
INFLUENCE OF RANDOM GAUSSIAN MEASUREMENT NOISE

Noise dB/°	**t mil**	ε_r	$tan\delta_\varepsilon$ 10^{-3}	σ, 10^{-2} *Si/m*	μ_r	$tan\delta_\mu$ 10^{-3}
Exact	**11**	**4.2**	**14**	**5**	**2.5**	**10**
0.1/3	10.87 ±0.33	4.196 ±0.16	12.0 ±12.2	5.2 ±3.9	2.56 ±0.14	24.9 ±14.2
0.2/6	11.09 ±0.44	4.06 ±0.31	24.3 ±20.9	4.3 ±3.3	2.67 ±0.28	19.0 ±18.5
0.4/12	11.39 ±0.77	3.98 ±0.31	24.0 ±21.4	7.3 ±3.9	2.58 ±0.40	14.8 ±16.1
0.8/12	11.04 ±0.76	4.34 ±0.33	15.8 ±21.7	4.8 ±4.2	2.24 ±0.19	19.6 ±20.6
Exact	**100**	**4.2**	**14**	**5**	**2.5**	**10**
0.1/3	101.1 ±1.2	4.170 ±0.038	13.5 ±2.3	4.64 ±0.95	2.477 ±0.040	11.1 ±2.4
0.2/6	101.2 ±4.3	4.17 ±0.12	15.6 ±5.1	4.7 ±1.7	2.47 ±0.10	8.26 ±4.7
0.4/12	103.7 ±5.5	4.08 ±0.18	14.5 ±4.5	5.2 ±2.4	2.44 ±0.12	8.29 ±3.2
0.8/12	95.3 ±8.4	4.35 ±0.23	11.2 ±7.2	5.1 ±2.8	2.61 ±0.25	9.98 ±6.6

C. *Decreasing the Impact of Measurement Noise Using Rolling Window Analysis*

To decrease the impact of random measurement noise, the use of rolling window (RW) analysis [8] was investigated. The principal tradeoff of this analysis is the size of RW in frequency domain that introduces the systemic error for any nonlinear TL curves.

TABLE V
SYSTEMIC ERROR OF MATERIAL PARAMETER DUE TO THE SIZE OF RW

W/P %	**t mil**	ε_r	$tan\delta_\varepsilon$ 10^{-3}	σ, 10^{-2} *Si/m*	μ_r	$tan\delta_\mu$ 10^{-3}
Exact	**100**	**4.2**	**14**	**5**	**2.5**	**10**
10	100.56	4.175	13.946	5.032	2.489	10.061
20	100.63	4.158	13.990	5.224	2.494	10.051
40	100.90	4.094	13.832	6.086	2.518	10.127
80	99.28	3.944	13.510	9.355	2.672	10.204

The systemic error of extracted material parameters as a function of the ratio between the RW size and the TL curve period (W/P) is illustrated in Table V, where data were calculated without the presence of measurement noise and for a thick sample only. For a thin sample with virtually linear TL characteristics no meaningful influence of RW size was observed. As is seen from Table V, the RW values up to 20% of the W/P do not introduce any significant systemic error.

The results of the application of RW analysis for noisy measurements are shown in Table VI, where data were calculated using W/P = 20%. The comparison between Tables IV and VI shows that the RW averaging helps to reduce the standard deviation of the extracted material parameters. However, for mean values of material parameters better results are achieved only for low level of measurement noise and mostly for "weak" material parameter values. Otherwise, the application of RW averaging either makes no difference, or even makes results somewhat worse.

This rather unexpected small effect of the RW averaging means that most averaging is effectively produced during the best-fitting of the wideband transmission loss curve.

TABLE VI
REDUCING INFLUENCE OF MEASUREMENT NOISE USING RW ANALYSIS

Noise dB/°	**t mil**	ε_r	$tan\delta_\varepsilon$ 10^{-3}	σ, 10^{-2} *Si/m*	μ_r	$tan\delta_\mu$ 10^{-3}
Exact	**11**	**4.2**	**14**	**5**	**2.5**	**10**
0.1/3	10.89 ±0.31	4.23 ±0.12	10.4 ±7.7	5.2 ±1.8	2.506 ±0.054	17.0 ±3.0
0.2/6	10.77 ±0.40	4.199 ±0.12	18.9 ±14.1	3.9 ±2.8	2.61 ±0.16	21.5 ±9.1
0.4/12	10.81 ±0.36	4.22 ±0.14	18.4 ±14.7	4.1 ±2.6	2.60 ±0.21	14.2 ±15.6
0.8/12	10.97 ±0.50	4.194 ±0.22	19.2 ±20.2	3.5 ±3.0	2.495 ±0.23	10.6 ±14.6
Exact	**100**	**4.2**	**14**	**5**	**2.5**	**10**
0.1/3	102.85 ±0.44	4.077 ±0.016	13.9 ±1.2	5.08 ±0.20	2.441 ±0.012	10.2 ±1.0
0.2/6	103.1 ±1.3	4.061 ±0.047	14.6 ±2.2	5.32 ±0.19	2.438 ±0.025	9.1 ±2.2
0.4/12	103.5 ±1.6	4.040 ±0.048	13.1 ±3.0	5.29 ±0.54	2.429 ±0.034	10.4 ±3.1
0.8/12	103.7 ±1.8	4.032 ±0.054	16.4 ±5.3	5.5 ±1.8	2.431 ±0.046	7.4 ±5.1

IV. NOVEL MEASUREMENT APPROACH FOR SAMPLES WITH VERY LOW LOSS WITHIN TEST FREQUENCY BAND

Here we introduce a novel approach for measurement of the CMD material parameters when the tested samples have low TL that are close or even below the measurement noise level. This could happen for two reasons. First, the sample might be thin enough to have TL characteristics within the test frequency band that are low in value and almost linear in shape. Second, the sample might be thick enough to exhibit the periodic behavior of the TL curves but has such a low conductivity, permittivity, and permeability that the absolute values of the TL are still close or even below the noise level.

In those cases, to increase the signal to noise ratio and enhance the tolerance of extracted CMD material parameters, two or more equidistantly spaced identical samples might be tested as one sandwich. The distance between samples should be chosen to deliver multiple minima of TL curve within the test frequency band. The number of samples should be enough to obtain the minima of TL curve that would be significantly above of the measurement noise level.

A. *Thin Sample Case*

The difference that makes the introduced "sandwich" measurement approach for thin CMD sample is illustrated in Table VII. Comparison between data in Tables IV and VII shows that the sandwich approach delivers better mean and tolerance values of extracted material parameters, especially for the "weak" ones such as conductivity and loss tangents.

TABLE VII
RESULTS FOR TWO THIN SAMPLES PLACED AT 1 INCH APART

Noise dB/°	**t mil**	ε_r	$\tan\delta_\varepsilon$ 10^{-3}	σ, 10^{-2} *Si/m*	μ_r	$\tan\delta_\mu$ 10^{-3}
Exact	**11**	**4.2**	**14**	**5**	**2.5**	**10**
0.1/3	11.04 ±0.15	4.184 ±0.047	14.5 ±1.7	4.83 ±0.44	2.503 ±0.024	10.6 ±3.1
0.2/6	10.91 ±0.20	4.219 ±0.066	14.3 ±3.2	5.05 ±0.74	2.520 ±0.034	11.6 ±5.0
0.4/12	11.22 ±0.32	4.144 ±0.096	13.6 ±3.3	5.31 ±1.2	2.475 ±0.064	8.8 ±6.1
0.8/12	11.06 ±0.60	4.205 ±0.18	7.2 ±6.9	6.8 ±1.9	2.477 ±0.092	15.6 ±14.5

B. *Thick Sample Case*

The difference that makes the "sandwich" measurement approach for thick CMD sample is illustrated in Table VIII. In the upper half of Table VIII, the material parameters were extracted based on one sample, while in the lower half of Table VIII the material parameters were extracted based on four consecutive samples placed at 0.1 inch apart.

TABLE VIII
RESULTS FOR ONE AND FOUR THICK SAMPLES PLACED AT 0.1 INCH APART

Noise dB/°	**t mil**	ε_r	$\tan\delta_\varepsilon$ 10^{-3}	σ, 10^{-3} *Si/m*	μ_r	$\tan\delta_\mu$ 10^{-3}
Exact	**400**	**1.8**	**1.4**	**2**	**1.3**	**1.0**
0.1/3	402.46 ±0.36	1.7882 ±0.003	1.313 ±0.24	2.153 ±0.31	1.2949 ±0.002	1.134 ±0.28
0.2/6	403.09 ±1.6	1.7891 ±0.005	1.240 ±0.35	2.221 ±0.95	1.2923 ±0.003	1.099 ±0.29
0.4/12	403.83 ±2.9	1.7844 ±0.014	1.258 ±0.35	2.043 ±1.6	1.2948 ±0.011	1.205 ±0.44
0.8/12	401.49 ±5.8	1.7913 ±0.026	1.379 ±0.86	1.581 ±1.9	1.2952 ±0.021	0.969 ±1.1
Exact	**400**	**1.8**	**1.4**	**2**	**1.3**	**1.0**
0.1/3	399.85 ±0.063	1.7973 ±.0005	1.222 ±0.28	2.190 ±0.30	1.3029 ±.0003	1.183 ±0.35
0.2/6	399.71 ±0.061	1.7958 ±.0012	1.255 ±0.22	2.348 ±0.39	1.3047 ±.0010	1.108 ±0.32
0.4/12	399.51 ±0.038	1.7952 ±.0009	1.252 ±0.28	2.127 ±0.34	1.3056 ±.0006	1.165 ±0.28
0.8/12	400.17 ±0.081	1.8006 ±.0013	1.431 ±0.35	2.266 ±0.47	1.2997 ±.0013	0.900 ±0.24

Comparison between upper and lower halves of Table VIII shows that as opposed to the thin sample case, for the thick samples the "sandwich" approach delivers slightly better results for a "strong" material parameters such as thickness, and the real part of permittivity and permeability, but does not help to determine a "weak" material parameters such as a specific conductivity and loss tangents any better than just using single sample measurement approach.

V. CONCLUSIONS

1\. The wideband free space TL only measurement technique can be used for the unique determination of homogeneous CMD material parameters, namely complex permittivity, permeability, and specific conductivity.

2\. Since only one TE or TM component of S_{21} element of the scattering matrix should be measured, this technique saves a considerable amount of test time (up to 85%) including time for the post-processing.

3\. Unlike the reflection loss based measurements, the TL measurements are not sensitive to the positioning of the material sample with respect to the transmit and receive horns.

4\. For unconditional convergence of the best-fitting procedure, the consecutive application of GA and DFP minimization algorithms can be used. Due to the significant material parameter tradeoffs, statistical analysis of the results must be applied.

4\. The wideband measurement allows greatly decrease the influence of the measurement noise on the values of the extracted material parameters.

5\. The application of RW averaging allows decrease the standard deviation of the extracted material parameters as well as decrease the lowest level of sample TL when the extraction of "weak" material parameters is still possible.

6\. For the case when the tested sample TL curve is close to linear and/or is comparable with the measurement noise, two or more properly spaced identical samples can be used for more reliable determination of the CMD material parameters.

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