Extended Source Size Correction Factor in Antenna Gain Measurements

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***Abstract*— In this paper we present improved formulations for the extended source size correction factors that are widely utilized in circular aperture reflector antenna gain measurements. Extended radio sources having an angular size that is comparable or larger than the FWHP antenna beamwidth are often used to determine the directivity of the antenna aperture. The resulting directivity measurements must be corrected to account for the convolution of the extended source angular size with the antenna's far field pattern beamwidth. Two kinds of extended radio sources, having either uniform or Gaussian brightness distributions over the source disk, as well as three kinds of the antenna aperture distributions: "Polynomial-on-Pedestal", Gaussian, and Taylor have been considered. The existing approximate formulae for the extended source size correction factor are examined and compared to the improved formulations.**

I. INTRODUCTION

It is well known that (see [1], [2]) most cosmic extended radio sources have either Gaussian or uniform brightness distributions over the source solid angle. The purpose of this paper is to approximate the Extended Source Size Correction Factor for cosmic extended radio sources with finite angular size, such as the sun, moon and planets, which are widely used for antenna directivity measurements on large diameter circular aperture reflector antennas.

In the case when the radio source angular size is comparable with the antenna FWHP beamwidth, the measured antenna directivity value is averaged within the solid angle of the source, and therefore, appears to be less than what would be expected for the antenna's effective collecting area and aperture illumination taper. The result of averaging depends on two factors: the source angular brightness distribution and spatial extent as well as the shape of antenna far field power pattern.

The convolution of these extended radio sources with the antenna far field patterns are evaluated for three kinds of antenna aperture distributions: "Polynomial-on-Pedestal", Gaussian, and Taylor, where we have used the antenna aperture distribution taper as a parameter.

Because the Extended Source Size Correction Factor is a function of the ratio between the extended source size and the antenna's FWHP beamwidth, the value of the antenna's FWHP beamwidth as a function of the aperture illumination taper for all three kinds of aperture illumination is considered as a supplementary problem.

In order to obtain the antenna's actual directive gain, the source size correction factor K is used [3]:

$$K = \frac{\iint_{\Omega_S} B_s(\Omega)\, d\Omega}{\iint_{\Omega_S} B_s(\Omega) F_n(\Omega)\, d\Omega} \qquad (1)$$

where $B_s(\Omega)$ is the brightness distribution of the extended radio source, $F_n(\Omega)$ is the normalized antenna power pattern, such that at boresight $F_n(0) = 1$, and Ω_S is the solid angle subtended to the extended source. To obtain the correct antenna directive gain value, the directive gain value measured using the extended radio source should be multiplied by *K*. The curves and formulae for the extended source size correction factor versus the ratio of the source size or source HPBW over the antenna HPBW have been developed in [1], [4], [5]. For the case of a Gaussian source and a Gaussian antenna power pattern, [1] and [4] state the following value of *K*:

$$K = 1 + x^2 \qquad (2)$$

where *x* is the ratio of the extended source HPBW over the antenna HPBW. For the case of a uniform disk brightness distribution and a Gaussian antenna power pattern, [4] and [5] express *K* as:

$$K = \frac{(x/1.2)^2}{1 - \exp[-(x/1.2)^2]} \approx 1 + 0.347x^2 + 0.0402x^4 \qquad (3)$$

where *x* is the ratio of the disk source diameter over antenna HPBW. For the case of a uniform disk brightness distribution and an antenna power pattern that corresponds with a uniform antenna aperture distribution, [5] gives the value of *K* as:

$$K = \frac{(1.616x)^2}{4[1 - J_1^2(1.616x) - J_0^2(1.616x)]} \qquad (4)$$

where x is the ratio of the disk source diameter to the antenna HPBW.

There are three problems that are associated with the approximations (2) – (4). First, the approximations (2) – (4) are based on the shape of the antenna power pattern rather than on the shape of the antenna aperture distribution, which is defined by the aperture illumination function and its taper, not simply from the value of the antenna far field FWHP beamwidth. Secondly, in order to use equations (2) – (4), the antenna FWHP beamwidth as a function of aperture illumination function and its taper needs to be known. Third, not all combinations of aperture and extended source distributions used in practice are covered by the (2) – (4).

II. The Approximation Of FWHP BW Of Circular Antenna With "Polynomial-On-Pedestal", Gaussian, And Taylor Aperture Illuminations

For the purpose of this paper we will use the following forms of "Polynomial-on-Pedestal", Gaussian, and Taylor aperture distributions:

$$f_{Ap\,poly} = B + (1 - B)(1 - (\frac{2r}{d})^2)^2 \qquad (5)$$

$$f_{Ap\,Gauss} = \exp[-(\sqrt{-\ln B}\,\frac{2r}{d})^2] \qquad (6)$$

$$f_{Ap\,Taylor} = \frac{J_0[j\pi\beta\sqrt{(1-(\frac{2r}{d})^2}]}{J_0[j\pi\beta]} \qquad (7)$$

where r is the value of the radius vector from the center of the aperture, d is the aperture diameter, B is the aperture illumination taper ($0 \le B \le 1$), and β can be found from the equation:

$$J_0[j\pi\beta] = \frac{1}{B} \qquad (8)$$

According to [6], the antenna HPBW can be estimated through the simple formula:

$$HPBW = \alpha\frac{\lambda}{d} \qquad (9)$$

where λ is the wavelength, d is the antenna diameter and α is the coefficient in degrees that depends on the aperture illumination and taper. The rough estimation of α as a function of the antenna taper, without taking into account the specific antenna aperture illumination function, was given in [7]:

$$\alpha = 55.9486 + 1.05238 \times b \qquad (10)$$

where b is the absolute value of illumination taper in dB. The number of digits in (10) is misleading because the six digit computational accuracy implied by the equation can not be achieved regardless of the illumination function, when based solely on the value of illumination taper.

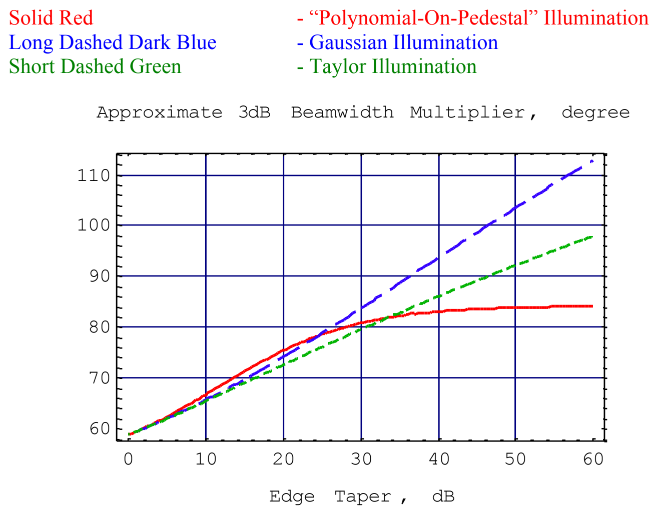

Fig. 1 Beamwidth Multiplier α for Formula (9) as a Function of Illumination Taper for Different Aperture Illumination Distributions (5) – (7)

Extensive numerical calculations of the antenna FWHP beamwidth for circular aperture reflector antennas were then conducted using the aperture distributions (5) – (7) as a function of illumination taper. These resulted in providing an improved approximation for the beamwidth multiplier coefficient α which is plotted in Fig. 1 for the three aperture illumination cases.

For the "Polynomial-on-Pedestal" illumination function with taper values from 0 to 20 dB and tolerance less than 0.7%, the beamwidth multiplier coefficient α can be approximated as follows:

$$\alpha = 58.862 + 0.7095 \times b + 0.0071 \times b^2 \qquad (11)$$

where b is the illumination taper in dB.

For the Gaussian illumination function, with taper values from 0 to 20dB and tolerance less than 0.1%, the beamwidth multiplier coefficient α can be approximated as follows:

$$\alpha = 58.862 + 0.601 \times b + 0.0084 \times b^2 \qquad (12)$$

For the Taylor illumination function, with taper values from 0 to 20dB and tolerance less than 0.3%, the beamwidth multiplier coefficient α can be approximated as follows:

$$\alpha = 58.862 + 0.603 \times b + 0.0048 \times b^2 \qquad (13)$$

III. Extended Source Size Correction Factor Approximation

A. Source with Uniform Disk Brightness Distribution

Using (1) for the calculations of the Extended Source Size Correction Factor and comparing the results with the approximate formulae (2) – (4) for the extended source with uniform disk brightness distribution shows that (4) gives the best approximation for all three considered type of aperture illuminations (5) – (7) at any illumination taper values. The fine difference between Extended Source Size Correction Factor given by (4) and the exact value calculated using (1) is shown in Figs. 2 – 4.

As can be seen from these plots, the error of the Extended Source Size Correction Factor given by (4) is very similar for all three types of aperture illuminations and is not more than 0.03 dB, when the source size is not greater than 150% of the antenna's FWHP beamwidth. When the source size is in the range of 150 to 300% of the antenna FWHP beamwidth, the error grows to 0.23 - 0.29 dB.

In order to utilize the results plotted in Figs. 2 – 4 for evaluation of the Extended Source Size Correction Factor when a radio source having a uniform brightness temperature over the solid angle is used, the Extended Source Size Correction Factor value should be computed by (4), converted into dB and the appropriate Correction Factor error for a given aperture illumination function and taper should be subtracted.

From inspection of the results, it is interesting to note that although equation (4) is theoretically valid for the uniform aperture illumination case only, it was found to describe the other considered aperture illuminations nearly as well. This means that the form of the aperture illumination does not make too much of a difference unless the source size becomes greater than 150% of the FWHP antenna beamwidth.

B. Source with Gaussian Brightness Distribution

Using (1) for the calculations of the Extended Source Size Correction Factor and comparing the results with approximate formulae (2) – (4) for the extended source with Gaussian brightness distribution shows that (2) gives the best approximation for all three considered types of aperture illumination (5) – (7) at any illumination taper values. The difference between the Extended Source Size Correction Factor given by (2) and the exact value calculated using (1) is shown in Figs. 5 – 7.

As can be seen from these plots, the error of the Extended Source Size Correction Factor given by (2) is very similar for all three types of aperture illumination and is not more than 0.03dB when the source size is less than 80% of the FWHP antenna beamwidth. When the source size is 80 to 300% of the FWHP antenna beamwidth, the error grows to 0.23 - 0.28 dB. In order to utilize the plots in Figs. 5 – 7 for evaluation of the Extended Source Size Correction Factor when a radio

Solid Red	- No Taper
Long Dashed Dark Blue	- 5dB Taper
Short Dashed Green	- 10dB Taper
Dot Dashed Light Blue	- 15dB Taper
Double Dot Dashed Yellow	- 20dB Taper
Triple Dot Dashed Purple	- 25dB Taper

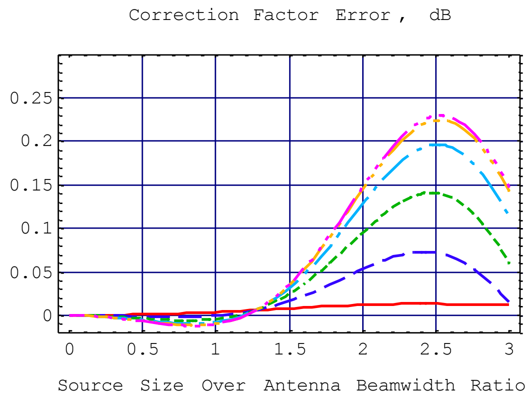

Fig. 2 Error of the Extended Source Size Correction Factor Calculated by (4) for Uniform Disk Brightness Distribution, "Polynomial-on-Pedestal" Aperture Illumination, and Different Illumination Tapers

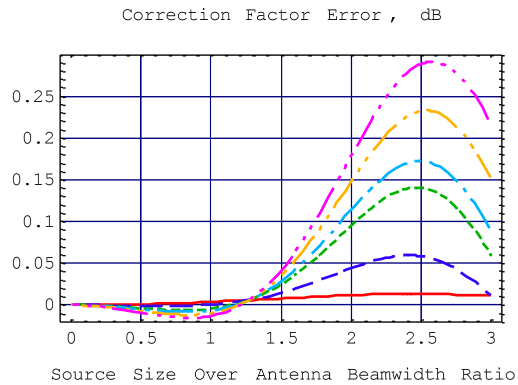

Fig. 3 Error of the Extended Source Size Correction Factor Calculated by (4) for Uniform Disk Brightness Distribution, Gaussian Aperture Illumination, and Different Illumination Tapers

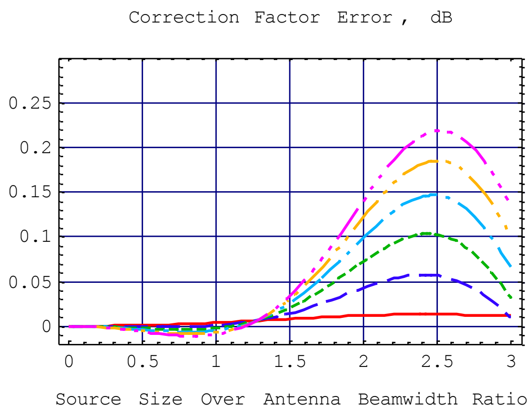

Fig. 4 Error of the Extended Source Size Correction Factor Calculated by (4) for Uniform Disk Brightness Distribution, Taylor Aperture Illumination, and Different Illumination Tapers

Solid Red - No Taper
Long Dashed Dark Blue - 5dB Taper
Short Dashed Green - 10dB Taper
Dot Dashed Light Blue - 15dB Taper
Double Dot Dashed Yellow - 20dB Taper
Triple Dot Dashed Purple - 25dB Taper

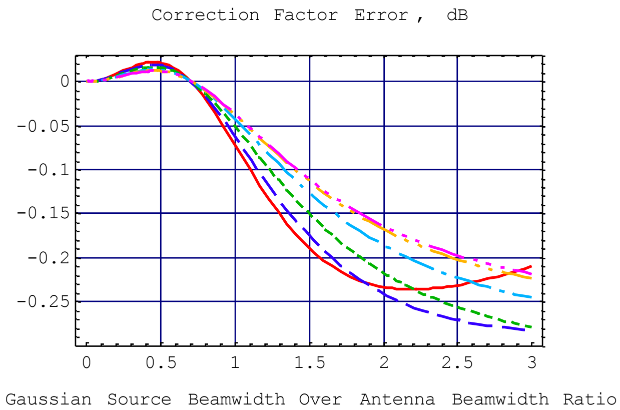

Fig. 5 Error of the Extended Source Size Correction Factor Calculated by (1) for Gaussian Source Brightness Distribution, "Polynomial-on-Pedestal" Aperture Illumination, and Different Illumination Tapers

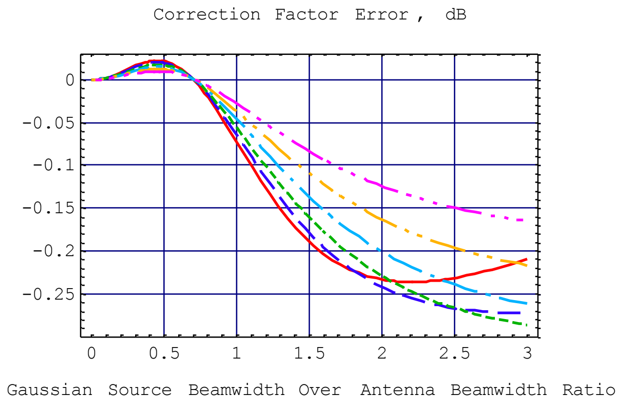

Fig. 6 Error of the Extended Source Size Correction Factor Calculated by (1) for Gaussian Source Brightness Distribution, Gaussian Aperture Illumination, and Different Illumination Tapers

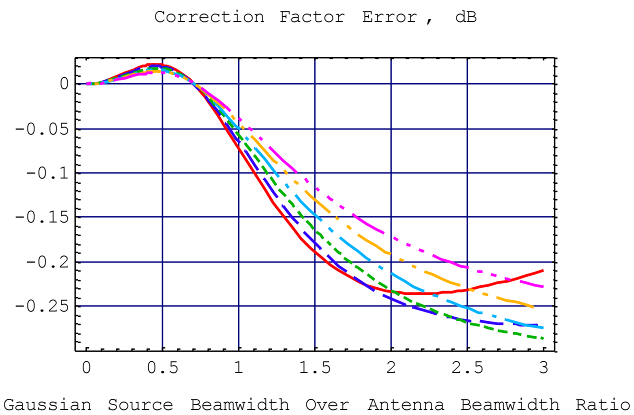

Fig. 7 Error of the Extended Source Size Correction Factor Calculated by (1) for Gaussian Source Brightness Distribution, Taylor Aperture Illumination, and Different Illumination Tapers

source having a Gaussian brightness distribution is used, its value should be computed by (2), converted into dB, and the Correction Factor error seen in the appropriate plot in Figs. 5 - 7 should be subtracted.

It's interesting to note that, in spite of the fact that equation (2) is theoretically valid only for Gaussian aperture illumination, it also describes all three considered aperture illuminations almost equally well, until the source size become more than 80% of the antenna FWHP beamwidth.

IV. Conclusions

Since extended cosmic radio sources are widely used for the measurement of large circular aperture reflector antenna directive gain, improvements in calibration accuracy are desirable. Correction factors for the subtended angle of the radio source as convolved with the antenna power pattern must be used to obtain valid directivity values, and this becomes even more important as the antenna size in wavelengths becomes very large. That is especially the case when very large diameter millimeter wavelength antennas are evaluated using extended cosmic radio sources.

This paper has developed a more accurate procedure for estimating the value of the Extended Source Size Correction Factor K without the need to perform the complicated numerical integration demanded by equation (1). Two types of extended source brightness distributions (uniform and Gaussian) along with three types of antenna aperture illuminations ("Polynomial-on-Pedestal", Gaussian, and Taylor) were considered. As a supplementary problem, the approximate formulae and plots for the antenna half power beamwidth for each of the above aperture illumination shapes with illumination tapers ranging from 0 to –60dB were obtained. Use of this improved approximate treatment will refine the values of directive gain as compared to the conventional approximations, especially where large aperture millimeter wave antennas are calibrated using the sun, moon or planets.

Cleared for Public Release by DoD/OSR on June 9, 2008, reference number 08-S-1751.

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